All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated. A scientific calculator is allowed.

Student Name: KEY **ID:**_____
Instructor: Mundy-Castle **Exam Score:** _____

1. Find the first derivative of each function.

a) $y = \text{arcsec}(x^2 - 2)$

$$\frac{dy}{dx} = \frac{2x}{|x^2-2|\sqrt{(x^2-2)^2-1}} = \frac{2x}{|x^2-2|\sqrt{x^4-4x^2+3}}$$

b) $y = x\left(4^{-3x}\right)$

$$\frac{dy}{dx} = 4^{-3x} + x\left(4^{-3x}\right)(\ln 4)(-3)$$

$$= 4^{-3x} - 3x \ln 4 \left(4^{-3x}\right)$$

c) $y = \ln(\cosh x)$

$$\frac{dy}{dx} = \frac{\sinh x}{\cosh x} = \tanh x$$

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2. Find the indicated integral.

a) $\int \frac{dx}{3+25x^2}$ $a=\sqrt{3}$, $u=5x$, $du=5dx$, $\frac{1}{5}du=dx$

$$\frac{1}{5}\int \frac{du}{3+u^2} = \frac{1}{5}\left(\frac{1}{\sqrt{3}}\right)\arctan\frac{u}{\sqrt{3}}+C = \frac{1}{5\sqrt{3}}\arctan\left(\frac{5x}{\sqrt{3}}\right)+C$$

b) $\int \sinh 6x dx$ $u=6x$, $du=6dx \rightarrow \frac{1}{6}du=dx$

$$\frac{1}{6}\int \sinh u\, du = \frac{1}{6}\cosh u + C = \frac{1}{6}\cosh 6x + C$$

c) $\int \sin^2 x \cos x dx$ $u=\sin x$, $du=\cos x\, dx$

$$\int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3 x + C$$

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3. Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y=\frac{x(x-1)^{3/2}}{\sqrt{x+1}}$, $x>1$.

$$\ln y = \ln\left[\frac{x(x-1)^{3/2}}{(x+1)^{1/2}}\right]$$

$$\ln y = \ln x + \tfrac{3}{2}\ln(x-1) - \tfrac{1}{2}\ln(x+1)$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{3}{2}\left(\frac{1}{x-1}\right) - \frac{1}{2}\left(\frac{1}{x+1}\right)$$

$$y' = \frac{x(x-1)^{3/2}}{\sqrt{x+1}}\left[\frac{1}{x} + \frac{3}{2x-2} - \frac{1}{2x+2}\right]$$

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4. Find the area under the curve $y = x\sqrt[3]{x+1}$, bounded by $y = 0$, $x = 0$, and $x = 7$.

$$\int_0^7 x(x+1)^{1/3}\,dx \qquad u = x+1\,,\ du = dx$$

$$x = u-1$$

$$\int_1^8 (u-1)u^{1/3}\,du = \int_1^8 u^{4/3} - u^{1/3}\,du$$

$$= \frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3}\Big|_1^8$$

$$= \frac{3}{7}(8)^{7/3} - \frac{3}{4}(8)^{4/3} - \left[\frac{3}{7}(1)^{7/3} - \frac{3}{4}(1)^{4/3}\right]$$

$$= \frac{3}{7}(128) - \frac{3}{4}(16) - \frac{3}{7} + \frac{3}{4}$$

$$= \frac{1209}{28} \qquad (\simeq 43.179)$$